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296 (Incorrectly numbered 294). Proposed by JOHN JAMES QUINN, Ph. D., Scottdale, Pa.

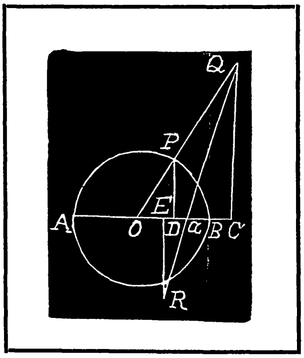
a) Suppose an indefinite line be pivoted at the end of a revolving radius whose center is the origin; and the initial position of the radius is coincident with the X-axis and the pivoted line perpendicular to it. As the radius revolves through equal amounts of arc the line moves to the right over corresponding equal intercepts on the X-axis. What is the equation of the locus of a point on the line whose distance from the end of the radius is equal to a diameter?

b) Show how the locus can be applied to the multisection of an angle.

c) Suppose the diameter be laid off in both directions.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let AB be a diameter of the given circle; $\angle POB=\theta=$angle OP makes with AB; a, the intersection of the line with AB; $PQ=PR=AB=2r$; $Ba=r\theta/90$; $Oa=r(1-\frac{1}{90}\theta)$; $Pa=r\sqrt{[1+(1-\frac{1}{90}\theta)^2-2(1-\frac{1}{90}\theta)\cos\theta]}=A$; $aQ=2r+A$; $aR=2r-A$; $PD=r\sin\theta$; $OD=r\cos\theta$.



$$QC=\frac{DP.Qa}{Pa}=\frac{r\sin\theta(2r+A)}{A}=y, \text{ for } Q \text{ locus.}$$

$$ER=\frac{r\sin\theta(2r-A)}{A}=y, \text{ for } R \text{ locus.}$$

$$aC=\frac{r}{A}(2r+A)(\cos\theta-1+\frac{\theta}{90}).$$

$$aE=\frac{r}{A}(2r-A)(\cos\theta-1+\frac{\theta}{90}).$$

$$OC=\frac{r(2r+A)\cos\theta-2r^2(1-\frac{1}{90}\theta)}{A}=x, \text{for} Q \text{locus.}$$

$$OE=\frac{2r^2(1-\frac{1}{90}\theta)-r(2r-A)\cos\theta}{A}=x, \text{ for } R \text{ locus.}$$

$$\therefore\ \rho^2=x^2+y^2=5r^2\pm\frac{4r^3-4r^3\cos\theta(1-\frac{1}{90}\theta)}{A}.$$

The plus sign is for Q, the minus for R. The same curve will be generated in the opposite way for the diameter (part) to the left of O.

Let ϕ be the angle to be multisected. Then $r\phi/90$ is the part of BO cut off by the pivoted line. Let $\theta=\phi/m$; this value of θ gives Pa. Let $Ba=r\phi/90m$. With a as center and Pa as radius, intersect the circle at P. Draw OP. Then POB is the required angle, ϕ/m. So on for any section or any number of sections.

299. Proposed by G. W. GREENWOOD, M. A., Dunbar, Pa.

Show that the circle on any focal radius of an ellipse touches the auxiliary circle.

Solution by B. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

Taking the left hand focus of ellipse as origin of rectangular coördinates, the axis of abscissas coinciding with the major axis of the ellipse, we have for the equation of the ellipse, auxiliar circle, and radius vector, respectively

$$\frac{(x-ae)^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1\ldots(1);\quad (x-ae)^2+y^2=a^3\ldots(2);\quad y=mx\ldots(3).$$

Solving (1) and (3), we find $x=\dfrac{a(1-e^2)[e\pm\sqrt{(1+m^2)}]}{1-e^2+m^2}=k$, say. Then $y=mk$. Hence, the coördinates of the center of the circle described on the focal radius are $\frac{1}{2}k$, $\frac{1}{2}mk$, and the equation of this circle is

$$(x-\tfrac{1}{2}k)^2+(y-\tfrac{1}{2}mk)^2=\tfrac{1}{4}k^2(1+m^2)\ldots(4).$$

Solving (2) and (4), we find that the discriminant of the resulting quadratic equations vanish, showing that the roots are equal. Hence, (4) intersects (2) in two coincident points and is, therefore, tangent to it.

CALCULUS.

222 (Incorrectly numbered 221, p. 117). Proposed by Professor F. ANDEREGG, Oberlin College, Oberlin, O.

If a, b, c, ... represent all the prime numbers 2, 3, 5, ... prove that

$$\left(1+\frac{1}{a^2}\right)\left(1+\frac{1}{b^2}\right)\left(1+\frac{1}{c^2}\right)\ldots\ldots=\frac{15}{\pi^2}.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\left(1+\frac{1}{2^2}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{5^2}\right)\left(1+\frac{1}{7^2}\right)\ldots\ \Big)$$

$$=\frac{\left(1-\frac{1}{2^4}\right)\left(1-\frac{1}{3^4}\right)\left(1-\frac{1}{5^4}\right)\left(1-\frac{1}{7^4}\right)\ldots}{\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{7^2}\right)\ldots}=\frac{\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\ldots}{\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+\ldots}$$

$$=\frac{\pi^6/6}{\pi^4/90}=\frac{15}{\pi^2}.$$

(See Vol. V, No. 5, May, 1898, page 134, third line from bottom, together with what precedes, for the complete solution. Also see Vol. XIII, No. 2, February, 1906, pages 40—41.)

223. Proposed by O. E. GLENN, Ph. D., Philadelphia, Pa.

Prove that $\displaystyle \mathop{\text{Lim.}}_{n\doteq\infty}\frac{\sum\limits_{\lambda=1}^{n}\lambda^k}{n^{k+1}}=\frac{1}{k+1}.$